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UNIT - 2

Axiomatic Definition Of Probability

Field of Events (F).

Let S be the Sample Space of a random experiment then the collection or class of sets F is called a field if it satisfies the following condition

(1) F is not empty

(2) The elements of F are Subsets of S.

(3) If $A \in F$. Then $A^c \in F$.

(4) if $A \in F$ and $C \in F$. then $A \cup C \in F$.

eg:- let $S = \{1, 2, 3, 4, 5, 6\}$ choose F as the Set with elements ϕ, S, $\{5, 6\}$ $\{1, 2, 3, 4\}$. Then F satisfies all the 4 conditions. So F is a field.

more generally when $A \subset S$. Then

$F = \{\phi, A, A^c, S\}$. forms a field.

Sigma field

Let S be a non empty Set and F be a collection of Subsets of S. Then F is called a σ - field if

(1) F is non empty
(2) the elements of F are subsets of S.
(3) if $A \in F$ then $A^c \in F$
(4) The union of any countable collections of elements of F is an element of F.

if $A_i \in F$, $i = 1, 2, \ldots n$. $A \cup C \in F$.
Then $\bigcup_{i=1}^{n} A_i \in F$
Then σ field F is also called Borel field and is often denoted by B.

Axiomatic definition of Probability

Let S be the sample space. Let B be the class of events constituting the Borel field. Then each $A \in B$. We can find a real valued set function P(A). known as the probability for occurance of A. if P(A) satisfies the following axioms

Axiom 1 (Non Negativity).
$0 \leq P(A) \leq 1$ for each $A \in B$

Axiom 2 (Norming)

Axiom 3 (Countable additivity)
if A. $A_1, A_2 . . . A_n$ is a finite or infinite
Sequence of elements in B such that $A_i \cap A_j = \phi$
, $i \neq j$.
Then Probability of $P(\bigcup_{i=1}^{n} A_i) = \sum_{i=1}^{n} P(A_i)$.

Theorems In Probability

Theorem 1

The Probability of an impossible ~~and~~ event is
zero ie $P(\phi) = 0$.

Proof.

let ϕ be the impossible event, then
$S \in B$. e $\phi \in B$.

Then we have $S \cup \phi = S$

$\Rightarrow P(S \cup \phi) = P(S)$.

$P(S) + P(\phi) = P(S)$. by axiom 3.

$1 + P(\phi) = 1$ by axiom 2 $P(S) = 1$.

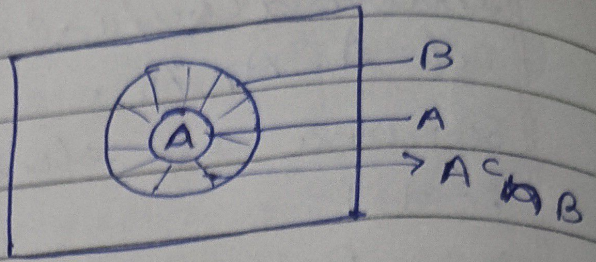
$P(\phi) = 1 - 1$

$P(\phi) = 0$

Theorem 2

if $A \subset B$ Then $P(A) \leq P(B)$.

from the venn diagram we have



$B = A \cup (A^c \cap B)$

$P(B) = P[A \cup (A^c \cap B)]$

$= P(A) + P(A^c \cap B)$

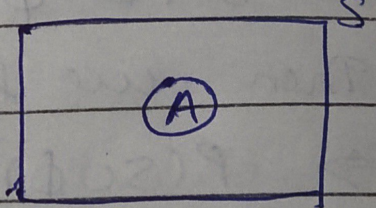
$\Rightarrow P(B) \geq P(A)$.

$\Rightarrow P(A) \leq P(B)$.

Theoram 3

Probability of A^c is $1 - P(A)$

$P(A^c) = 1 - P(A)$.



from venn diagram

$S = A \cup A^c$

$P(S) = P(A \cup A^c)$ Axiom 3

$P(S) = P(A) + P(A^c)$. Axiom 2

$1 = P(A) + P(A^c)$.

$1 - P(A) = P(A^c)$

$\Rightarrow P(A^c) = 1 - P(A)$.

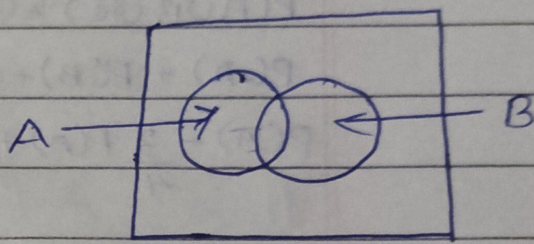
Theorem 4.

(1) $P(A \cap B^c) = P(A) - P(A \cap B)$.

(2) $P(A^c \cap B) = P(B) - P(A \cap B)$.

Proof.

if S is the Samplespace and A, B is the Subset of S. Then.



(1) from venn diagram

$A = (A \cap B^c) \cup (A \cap B)$.

$P(A) = P[(A \cap B^c) \cup (A \cap B)]$

$P(A) = P(A \cap B^c) + P(A \cap B)$.

$P(A) - P(A \cap B) = P(A \cap B^c)$.

$\Longrightarrow P(A \cap B^c) = P(A) - P(A \cap B)$.

(2) from venn diagram

$B = (A^c \cap B) \cup (A \cap B)$.

$P(B) = P[(A^c \cap B) \cup (A \cap B)]$

$P(B) = P(A^c \cap B) + P(A \cap B)$.

$P(B) - P(A \cap B) = P(A^c \cap B)$.

$\Longrightarrow P(A^c \cap B) = P(B) - P(A \cap B)$.

Q) A, B, C are three mutually exclusive and exhaustive events associated with a random experiment. find P(A). given that $P(B) = \frac{3}{4} P(A)$ and $P(C) = \frac{1}{3} P(B)$.

$A \cup B \cup C = S$. exhaustive events.

$P(A \cup B \cup C) = P(S)$.

$P(A) + P(B) + P(C) = P(S)$.

$P(A) + \frac{3}{4} P(A) + \frac{1}{3} \cdot \frac{3}{4} P(A) = 1$

$P(A) + \frac{3}{4} P(A) + \frac{1}{4} P(A) = 1$.

$1P(A) + \frac{3+1}{4} P(A) = 1$.

$2P(A) = 1$

$P(A) = \frac{1}{2}$.

Q. if two dice are thrown what is the probability that sum is.

(1) greater than 9.

(2) neither 10 nor 12.

Ans. (1) $A = \{(4,6)\ (5,5)\ (5,6)\ (6,4)\ (6,5)\ (6,6)\}$.

$P(A) = \frac{6}{36}$

(ii) B = Sum is 10 = {(5,5) (4,6) (6,4)}

C = Sum is 12 = {(6,6)}

$P(\text{neither 10 nor 12}) = P(\bar{B} \cap \bar{C})$

$= P(\overline{B \cup C})$

$= 1 - P(B \cup C)$

$= 1 - [P(B) + P(C)]$

$= 1 - P(B) - P(C).$

$P(B) = \frac{3}{36}$ $\quad = 1 - \frac{3}{36} - \frac{1}{36}$

$P(C) = \frac{1}{36}$ $\quad = 1 - \frac{4}{36}$ $\quad \frac{1}{9}.$

$= 1 - \frac{1}{9}$

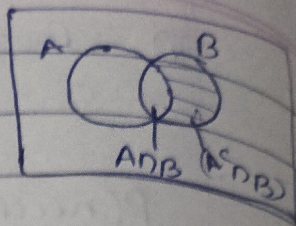
$= \frac{8}{9}.$

Theoram 5

Additional Theoram of Probability of two events

Let S be the Sample space of random experiment and events A & B is a subset of Sample space. Then $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.

from venn diagram.



Let $A \cup B = A \cup (A^c \cap B)$.

~~$P(A \cup B) = P(A) + P(A^c \cap B)$.~~

$$P(A \cup B) = P[A \cup (A^c \cap B)]$$

$$= P(A) + P(A^c \cap B). \quad \text{from theorem}$$

$$= P(A) + P(B) - P(A \cap B)$$

Theorem 6

Additional Theorem for three events

If A, B, C are three events then probability of $A \cup B \cup C$ is.

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C).$$

Let $D = B \cup C$.

$$P(A \cup B \cup C) = P(A \cup D). \qquad A \cup A^c \cap D.$$

$$= P(A) + P(B) - P(A \cap D).$$

$$= P(A) + P(B \cup C) - P(A \cap (B \cup C)). \rightarrow$$

$$= P(A) + P(B) + P(C) - P(B \cap C) - P[(A \cap B) \cup (A \cap C)]$$

$$= P(A) + P(B) + P(C) - P(B \cap C) - [P(A \cap B) + P(A \cap C) - P[A \cap B \cap C]]$$

$$= P(A) + P(B) + P(C) - P(B \cap C) - P(A \cap B) - P(A \cap C) + P(A \cap B \cap C).$$

am 4.

Q) If two dice are thrown what is the probability that the sum is
1. greater than 8.
2. neither 7 nor 11.

Ans) Total possible events $= 6^2 = 36$.

- greater than 8:

$A = \{(4,5)\ (4,6)\ (5,5)\ (6,3)\ (6,4)\ (5,4)\ (6,5)\ (5,6)\ (6,6), (3,6)\}$

$$P(A) = \frac{10}{36} = \frac{5}{18}$$ //.

- neither 7 nor 11

$P(B) \rightarrow$ Sum 7 or Sum 11.

$P(C) \Rightarrow$ Sum neither than 7 or Sum 11 $\Rightarrow 1 - P(B)$.

$B = \{(1,6)\ (2,5)\ (3,4)\ (4,3)\ (5,2)\ (1,6)\ (5,6)\ (6,5)\}$.

$$P(B) = \frac{8}{36}.$$

$$\begin{array}{r} \overset{2}{3}6 \\ -\ 8 \\ \hline 28 \end{array}$$

$$P(C) = 1 - \frac{8}{36}$$

$$= \frac{36-8}{36} = \underline{\underline{\frac{28}{36}}} = \underline{\underline{\frac{7}{9}}}.$$

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Q) Two dice are tossed. find the probability of getting an even number on a first die or a total of 8.

Ans) A → Sum 8.

B → even no ~~of~~ on 1st die

$A = \{(6,2)\ (5,3)\ (4,4)\ (3,5)\ (2,6)\}$

$B = \{(2,1)\ (2,2)\ (2,3)\ (2,4)\ (2,5)\ (2,6)$
$(4,1)\ (4,2)\ (4,3)\ (4,4)\ (4,5)\ (4,6)$
$(6,1)\ (6,2)\ (6,3)\ (6,4)\ (6,5)\ (6,6)\}$

$P(A) = \frac{5}{36}$

$A \cap B = \{(6,2)\ (4,4)\ (2,6)\}$

$P(B) = \frac{18}{36}$

$P(A \cap B) = \frac{3}{36}$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{5}{36} + \frac{18}{36} - \frac{3}{36}$

$= \frac{20}{36}$

$= \frac{5}{9}$

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Q) An MBA ~~applicant~~ applies ~~applies~~ for a job in terms of x and y. an
The prob of his being Selected in firm is 0.7.
And being rejected at y is 0.5.
The probability of atleast one of his application is rejected is 0.6. What is the probability that he will be selected in one of the firms.

A => Selected in X
B => Selecting in Y.

$P(A) = 0.7$

$P(\bar{B}) = 0.5$

$P(\bar{A} \cup \bar{B}) = 0.6$

$P(A \cup B) = ?$

$P(\bar{A} \cup \bar{B}) = P(\overline{A \cap B}) = 0.6$

$P(A \cap B) = 1 - 0.6$

$= \underline{0.4}$

$P(\bar{B}) = 0.5$

$P(B) = 1 - 0.5 = 0.5$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.7 + 0.5 - 0.4$

$= 0.7 + 0.1$

$= \underline{\underline{0.8}}$

8)

Ans: $E \rightarrow$ A $\Rightarrow P(E) = \frac{20}{100}$

$F \rightarrow$ B $\Rightarrow P(F) = \frac{16}{100}$

$G \rightarrow$ C $\Rightarrow P(G) = \frac{14}{100}$

$P(E \cap F) = \frac{8}{100}$

$P(E \cap G) = \frac{5}{100}$

$P(F \cap G) = \frac{4}{100}$

$P(E \cap F \cap G) = \frac{2}{10000}$

$P(E \cup F \cup G) = P(E) + P(F) + P(G) -$

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Q) 3 news paper A, B & C are published in a certain city, it is ~~established~~ estimated from a survey that of the adult population 20% read A, 16% read B, 14% read C, 8% read both A & B, 5% read Both A & C, 4% read both B & C, 2% read all three. Find what percentage read atleast one of the papers

Ans: E → A ⇒ $P(E) = \frac{20}{100}$

F → B ⇒ $P(F) = \frac{16}{100}$

G → C ⇒ $P(G) = \frac{14}{100}$

$P(E \cap F) = \frac{8}{100}$

$P(E \cap G) = \frac{5}{100}$

$P(F \cap G) = \frac{4}{100}$

$P(E \cap F \cap G) = \frac{2}{10000}$

$P(E \cup F \cup G) = P(E) + P(F) + P(G) -$

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Q) Two unbaised dice are thrown once. find the probability that
(1) Sum of upper face of the dice is 10 or more or 1st dice shows no 6.
(2) 2nd dice shows 6. or sum of the upper face of the dice is less than 7

$\Rightarrow S = \{(1,1)\ (1,2)\ (1,3)\ (1,4)\ (1,5)\ (1,6)\ (2,1)\ (2,2)\ (2,3)\ (2,4)\ (2,5)\ (2,6)$
$(3,1)\ (3,2)\ (3,3)\ (3,4)\ (3,5)\ (3,6)\ (4,1)\ (4,2)\ (4,3)\ (4,4)\ (4,5)\ (4,6)$
$(5,1)\ (5,2)\ (5,3)\ (5,4)\ (5,5)\ (5,6)\ (6,1)\ (6,2)\ (6,3)\ (6,4)\ (6,5)\ (6,6)\}$

(1) A = Sum of upper face is 10 or more (6,4) (4,6) (5,5) (6,5) (5,6) (6,6)
B = 1st dice 6 $\Rightarrow$ 6.

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{6}{36} + \frac{6}{36} - \frac{3}{36}$

$= \frac{9}{36} = \frac{1}{4}$

(2) C $\Rightarrow$ 2nd dice 6 $\Rightarrow$ 6. $P(C) - 6/36$.

D $\Rightarrow$ Sum less than 7 $\rightarrow$ (1,5) (1,4) (1,3) (1,2) (1,1) (2,1) (2,2) (2,3) (2,4) (3,1) (3,2) (3,3) (4,1) (4,2) (5,1) $= \frac{15}{36}$.

$P(C \cup D) = P(C) + P(D)$

$= \frac{6}{36} + \frac{15}{36} = \frac{21}{36} = \frac{7}{12}$

Q) A

find P of getting a king or a heart or a red card.

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A → P of getting king $4/52$.
B → P of getting a heart $13/52$.
C → P of getting a red card $26/52$

$A \cap B = 1$ $A \cap C = 2$ $B \cap C = 13/52$. $A \cap B \cap C = 1$

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

$= \frac{4}{52} + \frac{13}{52} + \frac{26}{52} - \frac{1}{52} - \frac{2}{52} - \frac{13}{52} + \frac{1}{52}$.

$= \frac{28}{52} = \frac{7}{13}$

$\frac{30}{13}$

Q) A Survey is conducted over 100 persons living in a city centre and observed that among these 100 persons. 65 persons. reads paper A, 40 person read paper B and 70 persons read paper C. 30 person read both A & B and 40 person read both A & C. 25 read both B & C and 20 person read all 3 papers.

A person is selected at random. what Probability that

① He reads only A.

② He reads none of 3 papers.

③ He reads atleast one paper.

(4) he reads exactly 2 paper.

65

Ans. A → 65

B → 40

C → 70

$A \cap B \to 30$

$A \cap C \to 40$

$B \cap C \to 25$

$A \cap B \cap C \to 20$

$$\begin{array}{r} 70 \\ -20 \\ \hline 50 \end{array} \qquad \begin{array}{r} 65 \\ 50 \\ \hline 15 \end{array}$$

$P(A) - P(A \cap B)$

$P(A \cap \bar{B}) = P(A) + P(\bar{B}) - P(A \cup \bar{B})$

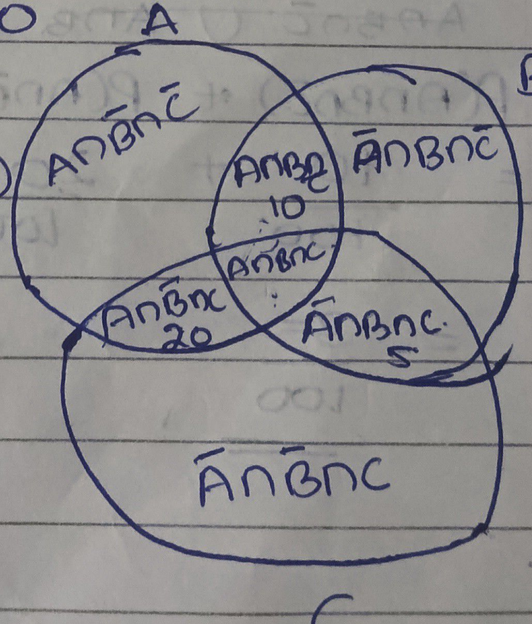
~~= P(A) + 1 - P(B) - P(A) + P(A ∩ B)~~

(1) $P(A \cap \bar{B} \cap \bar{C}) = P(A) - P(A \cap B) - P(A \cap C) + P(A \cap B \cap C)$

$$= \frac{65}{100} - \frac{30}{100} - \frac{40}{100} + \frac{20}{100}$$

$$= \frac{65}{100} - 50/100$$

$$= \underline{15/100}$$



$= P(A) - P(A \cap B) - P(A \cap \bar{B} \cap C)$

$$= \frac{65}{100} - \frac{30}{100} - \frac{20}{100}$$

$$= \frac{15}{100}$$

$$\begin{array}{r} 30 \\ 40 \\ 25 \\ \hline 95 \\ 60 \\ \hline 35 \end{array} \qquad \begin{array}{r} 20 \\ 3\times \end{array}$$

Q) From a pack of 52 cards. 1 card is taken at random. find the probability that the.

1) card is either an ace or spade.

2) is either a spade or heart.

Ans) total cases - 52.

1) A → ace. ⇒ 4.

B → Spade → 13.

$A \cap B \Rightarrow 1$.

$$P(A \cup B) = P(A) + P(B) - P(A \cap B).$$

$$= \frac{4}{52} + \frac{13}{52} - \frac{1}{52}$$

$$= \frac{16}{52}$$

2) C ⇒ spade → 13.

D ⇒ heart → 13.

$P(C \cap D) = 0$

$$P(C \cup D) = P(C) + P(D)$$

$$= \frac{13}{52} + \frac{13}{52}.$$

$$= \frac{26}{52}.$$

② $\overline{(A \cup B \cup C)}$

$P(\overline{A \cup B \cup C}) = 1 - P(A \cup B \cup C)$

$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$

$= \frac{65}{100} + \frac{40}{100} + \frac{70}{100} - \frac{30}{100} - \frac{40}{100} - \frac{25}{100} + \frac{20}{100}$

$= \frac{100}{100} = 1.$

$P(\overline{A \cup B \cup C}) = 1 - 1 = 0$

③ $P(A \cup B \cup C) = 1$

④ ~~$P((A \cap B) \cup (B \cap C) \cup (A \cap C))$.~~

~~$A \cap B \cap \bar{C} \cup A \cap B$~~ P(exactly two naners).

④ $P(A \cap B \cap \bar{C}) + P(A \cap \bar{B} \cap C) + P(\bar{A} \cap B \cap C)$.

$= \frac{10}{100} + \frac{20}{100} + \frac{5}{100}$

$= \frac{35}{100}$

Q) A Company produces spare parts using two machine A & B. In a day. Machine A & B produces 400 & 300 spare parts respectively. it is known that 10% of spare parts of A. and 20%. of B are usually find defective. from a days output a spare part is selected at random. what is the probability that it is a product of A or it is a deffective spare part.

Ans) $E_1 \Rightarrow$ product of A = 400

$E_2 \Rightarrow$ product of B = 300

$D_1 \rightarrow$ defective product of A. = $\frac{10}{100} \times 400 = 40$

$D_2 \rightarrow$ defective product of B. = $\frac{20}{100} \times 300 = 60$

$E_1 \cap D_1 = 40$ $\qquad D \Rightarrow 100$

$P(E_1 \cup D_1 \cup D_2)$ $\qquad P(E_1 \cup D) = P(E_1) + P(D) - P(E_1 \cap D)$

~~$D = D_1 \cup D_2$~~ $\qquad = \frac{400}{700} + \frac{100}{700} - \frac{40}{700}$

$= P(E_1) + P(D_1) + P(D_2) - P(E_1 \cap D_1) - P(E_1 \cap D_2) + P(D_1 \cap D_2) + P(E_1 \cap D_1 \cap D_2)$

$= \frac{400}{700} + \frac{40}{700} + \frac{60}{700} - \frac{40}{700} -$

$= \frac{460}{700} = \frac{46}{70} = \frac{23}{35}$

Ans) a) mutually exclusive $\rightarrow$ $P(A\cap B) = 0$

$P(A\cup B) = P(A) + P(B) - P(A\cap B)$

$P(B) = P(A\cup B) - P(A)$

$= 0.7 - 0.4$

$= \underline{\underline{0.3}}$

b) for independent events

$P(A\cap B) = P(A) \cdot P(B)$

$P(A) + P(B) - P(A\cup B) = P(A) \cdot P(B)$

$0.4 + P - 0.7 = 0.4 \cdot P$

$-0.3 + P = 0.4P$

$1 - 0.4P = 0.3$

$0.6\, P = 0.3$

$P = \dfrac{0.3}{0.6}$

$= 1/2 = \underline{\underline{0.5}}$

Q). What is the probability that a +ve integer not exceeding 50 selected at random is divisible by 3 or 5

Ans) A $\rightarrow$ divisible by 3. $\underline{16}$

3, 6, 9, 12, 15, 18, 21, 24, 27, 30, 33, 36, 39, 42, 45, 48

B $\rightarrow$ divisible by 5 $= 10$

5, 10, 15, 20, 25, 30, 35, 40, 45, 50

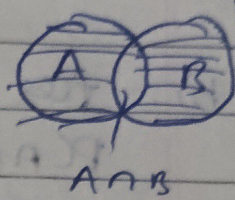
Q) if $P(A) = 0.4$, $P(B) = 0.7$ and
P[at least one of A & B] $= 0.8$ find
P[only one of A & B].

Ans) $P(A) = 0.4$ $\quad P(B) = 0.7$ $\quad P(A \cup B) = 0.8$

$P(A \cap B) = 0.4 + 0.7 - 0.8$

$= \cancel{1.1} - 0.8$

$= \underline{0.3}$.



1.1
-0.8
0.3

P[only one of A & B) $= P(A \cup B) - P(A \cap B)$

$= 0.8 - 0.3$

$= \underline{0.5}$.

$= P[(A \cap \bar{B}) \cup (\bar{A} \cup B)]$

$= P(A \cap \bar{B}) + P(\bar{A} \cup B)]$.

$= P(A) - P(A \cap B) + P(B) - P(A \cap B)$.

$= P(A) + P(B) - 2P(A \cap B)$.

$\Rightarrow P(A \cap B) = P(A) + P(B) - P(A \cap B)$.

Q) Let A and B The possible outcomes and Suppose $P(A) = 0.4$, $P(A \cup B) = 0.7$, and $P(B) = p$.

1. for what choise of P are A and B mutually exclusive.
2. for what choise ~~and~~ of B A and B independent
3.

$A \cap B = 3$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{16}{50} + \frac{10}{50} - \frac{3}{50}$

$= \frac{23}{50}$ // $= \frac{\cancel{}}{\cancel{20}}$ // $= \frac{23}{50}$

Q) What is the probability that a +ve integers from 1, 2, 3, ... 20. selected at random is divisible 3 or it is a prime no.

A ⟹ divisible by 3   3, 6, 9, 12, 15, 18 $= \frac{6}{20}$

B ⟹ prime   2, 3, 5, 7, 11, 13, 17, 19 $= \frac{8}{20}$

$A \cap B = 1$ $\frac{1}{20}$

$A \cup B = P(A) + P(B) - P(A \cap B)$

$= \frac{6}{20} + \frac{8}{20} - \frac{1}{20}$

$= \frac{13}{20}$ //

Ans (1) $P(\bar{A} \cap \bar{B}) = P(\overline{A \cup B})$

$= 1 - P(A \cup B)$

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= 0.30 + 0.78 - 0.16$

$= \underline{0.92}$

$P(\bar{A} \cap \bar{B}) = 1 - 0.92$

$= \underline{0.08}$

$$\begin{array}{r} 0.78 \\ -\,0.16 \\ \hline 0.62 \\ +\,0.30 \\ \hline 0.92 \end{array} \qquad \begin{array}{r} 1.00 \\ -\,0.92 \\ \hline 0.08 \end{array} \qquad \begin{array}{r} 0.92 \\ +\,0.08 \\ \hline 1.00 \end{array}$$

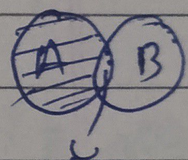
(2) $P(\bar{A} \cup \bar{B}) = P(\overline{A \cap B})$

$= 1 - P(A \cap B)$

$= 1 - 0.16$

$= \underline{0.84}$

$$\begin{array}{r} 1.00 \\ -\,0.16 \\ \hline 0.84 \end{array}$$



$A \cap B$

$A \cap \bar{B}$

(3) $P(A \cap \bar{B}) = P(A) - P(A \cap B)$

$= 0.30 - 0.16$

$= \underline{0.14}$

$$\begin{array}{r} 0.30 \\ -\,0.16 \\ \hline 0.14 \end{array}$$

Q) A husband and wife appears in an interview for 2 vacancy in a firm. The probability of selection of husband is $\frac{1}{7}$ and that of wifes selection is $\frac{1}{5}$. What is the probability that

① both of them will be selected

② Only one of them will be selected.

③ None of them will be selected.

Theoram Boole's Inequality

Let $A_1, A_2, \ldots A_n$ be n events defined on a Sample Space S of a random experiment. Then:

(1) $P(\bigcap_{i=1}^{n} A_i) \geq \sum_{i=1}^{n} P(A_i) - (n-1)$

(2) $P(\bigcup_{i=1}^{n} A_i) \leq \sum_{i=1}^{n} P(A_i)$.

Q) Let A and B be the events such that $P(A) = 0.7$ $P(B) = 0.5$. Show that $P(A \cap B) \geq 0.2$

Ans) $P(A \cap B) \geq P(A) + P(B) - (2-1)$

$\geq P(0.7 + 0.5 - 1$

$\geq 1.2 - 1$

≥ 0.2

Q) Given $P(A) = 0.30$ $P(B) = 0.78$ $P(A \cap B) = 0.16$ find

(I) $P(\bar{A} \cap \bar{B})$.

(II) $P(\bar{A} \cup \bar{B})$.

(III) $P(A \cap \bar{B})$.

Ans) A $\rightarrow$ ~~Hule~~ Husband is selected

B $\rightarrow$ wife is selected

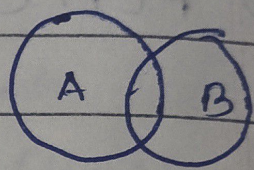
$P(A) = 1/7 \qquad P(B) = \frac{1}{5}$

a) Since it is independent events.

$$P(A\cap B) = P(A) \cdot P(B)$$

$$= \frac{1}{7} \times \frac{1}{5}$$

$$= \frac{1}{35}$$



b) $P((A\cap \bar{B}) \cup (\bar{A}\cap B))$

$$= P(A\cap \bar{B}) \cup P(\bar{A}\cap B)$$

$$= P(A) - P(A\cap B) + P(B) - P(A\cap B)$$

$$=$$

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

$$= \frac{1}{7} + \frac{1}{5} - \frac{1}{35}$$

$$= \frac{5}{35} + \frac{7}{35} - \frac{1}{35}$$

$$= \frac{11}{35}$$

$$P(A\cap\bar{B} \cup \bar{A}\cap B) = P(A\cup B) - P(A\cap B)$$
$$= \frac{11}{35} - \frac{1}{35}$$
$$= \frac{10}{35} = \frac{2}{7}.$$

c) $P(\bar{A}\cap\bar{B})$.
$$= P(\overline{A\cup B})$$
$$= 1 - P(A\cup B)$$
$$= 1 - \frac{11}{35}$$
$$= \frac{35-11}{36} = \frac{24}{36}.$$

Q) A problem in statistics given 3 students A, B, C whose chances of solving that 1/2, 3/4, 1/4 respectively. What is the probability that the problem will be solved.

Ans) $P(A) = 1/2$ $\quad P(B) = 3/4$ $\quad P(C) = 1/4$.

$P(A\cap B) = \frac{1}{2} \times \frac{3}{4} = \frac{3}{8}$ $\quad P(A\cap B\cap C) = \frac{1}{2} \times \frac{3}{4} \times \frac{1}{4}$

$P(B\cap C) = \frac{3}{4} \times \frac{1}{4} = \frac{3}{16}$

$P(A\cap C) = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$ $\quad = \frac{3}{32}$

$\cancel{P(A\cup B\cup C)} =$

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$$P(A\cup B\cup C) = P(A)+P(B)+P(C) - P(A\cap B) - P(B\cap C) - P(A\cap C) + P(A\cap B\cap C).$$

$$= \frac{1}{2} + \frac{3}{4} + \frac{1}{4} - \frac{3}{8} - \frac{3}{16} - \frac{1}{8} + \frac{3}{32}$$

$$= \frac{1\times16}{2\times16} + \frac{4\times8}{4\times8} - \frac{4\times4}{8\times4} - \frac{3\times2}{16\times2} + \frac{3}{32}$$

$$= \frac{16}{32} + \frac{32}{32} - \frac{16}{32} - \frac{6}{32} + \frac{3}{32}$$

$$\begin{array}{r} 32 \\ 16 \\ \hline 3 \end{array}$$

$$= \frac{32}{32} - \frac{3}{32}$$

$$= \frac{29}{32}.$$

$A\cap B\cap C$

$1 - \frac{3}{32}$

$$P(A\cup B\cup C) = P(\overline{A\cap B\cap C})$$

$$= 1 - P(\bar{\bar{A}}\cap B\cap C)$$

$$= 1 - \frac{3}{32}$$

$$= \frac{32-3}{32} = \frac{29}{32},$$

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Q) A Sales man has 60% chance of making a sale to each customer. If 2 customer A & B enter the shop find the probability that he will make ~~the~~ a sale to A or B. assuming that behaviour of the customer is independent

A → Sale to A $P(A) = 60/100$

B → Sale to B. $P(B) = 60/100$.

$P(A \cap B) = P(A) \times P(B)$.

$= \frac{60}{100} \times \frac{60}{100} = \frac{36\cancel{00}}{100\cancel{00}} = \frac{36}{100}$.

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$

$= \frac{60}{100} + \frac{60}{100} - \frac{36}{100}$

$$\begin{array}{r} 120 \\ 36 \\ \hline 84 \end{array}$$

$= \frac{84}{100}$

Q). Let E, F, G be mutually independant events $P(E) = 1/2$ $P(F) = 1/3$ $P(G) = 1/4$. Let (P) be the probability that atleast 2 of the events among E F G occur then $12 \times P$.

Ans). $P(E) = 1/2$ $P(F) = 1/3$ $P(G) = 1/4$.

$P(E \cap F) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$ $P(F \cap G) = \frac{1}{3} \times \frac{1}{4} = \frac{1}{12}$

$P(E \cap G) = \frac{1}{2} \times \frac{1}{4} = \frac{1}{8}$ $P(E \cap F \cap G) = \frac{1}{2} \times \frac{1}{3} \times \frac{1}{4} = \frac{1}{24}$

$$\frac{1}{4} = P(A) - P(A)\cdot P(B)$$

$$\frac{1}{4} = \left(1 - \frac{1}{4}\right) P(A).$$

$$\frac{1}{4} = \left(\frac{3}{4}\right) P(A).$$ $\frac{3}{4} P(A)$

$$\frac{1}{4} \times \frac{4}{3} = P(A).$$

$$\frac{1}{3} = P(A).$$

$$\frac{1}{2} = P(A) + \frac{1}{4} - P(A)\cdot\frac{1}{4}.$$

$$\frac{1}{2} - \frac{1}{4} = P(A) - P(A)\cdot\frac{1}{4}$$

$$\frac{1}{4} = \left(1 - \frac{1}{4}\right) P(A)$$

$$\frac{1}{4} \times \frac{4}{3} = \frac{3}{4} P(A).$$

$$\frac{1}{3} = P(A)$$

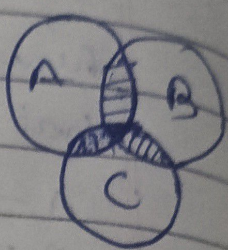
Q). Let E_1, E_2, & E_3 be three events such that $P(E_1 \cap E_2) = \frac{1}{4}$, $\cancel{P(E_1 \cap E_3)} =$ $P(E_1 \cap E_3) = P(E_2 \cap E_3) = \frac{1}{5}$; $P(E_1 \cap E_2 \cap E_3) =$ [illegible]. Then among the events E_1, E_2 & E_3. the probability that atleast two events. Occur equal.

A)

$P = P(E\cap F) + P(F\cap G) + P(E\cap G) - 2P(E\cap F\cap G)$

$= \frac{1}{6} + \frac{1}{12} + \frac{1}{8} - 2\times\frac{1}{24_{12}}$

$= \frac{1}{6} + \cancel{\frac{1}{12}} + \frac{1}{8} - \cancel{\frac{1}{12}}$



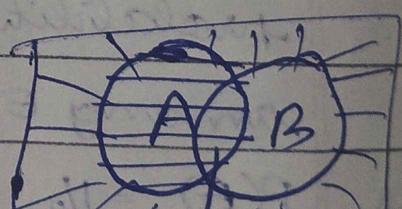
$= \frac{1}{6} + \frac{1}{8}$

$= \frac{1\times 4}{6\times 4} + \frac{1\times 3}{8\times 3}$

$= \frac{4+3}{24} = \frac{7}{24}$

$12P = \cancel{12}\times\frac{7}{\cancel{24}_{2}} = \frac{7}{2}$ //.

Q) Let A and B be two events such that $P(B) = 3/4$. $P(A\cup\bar{B}) = 1/2$. If A and B are independent then $P(A) = ?$



$A\cap B$

$P(B) = \frac{3}{4}$

$P(\bar{B}) = 1 - \frac{3}{4} = \frac{1}{4}$

$P(A\cup\bar{B}) = P(A) + P(\bar{B}) - P(A\cap\bar{B})$

$\frac{1}{2} = P(A) + \frac{1}{4} - P(A)\cdot P(\bar{B})$

Ans) ~~P(E∩~~

P(of atleast two events) =

~~P(E_1∩E_2) ∪ (E_2∩~~

$P(E_1 \cap E_2) + P(E_2 \cap E_3) + P(E_1 \cap E_3) - 2P(E_1 \cap E_2 \cap E_3)$

$= \frac{1 \times 5}{4 \times 5} + \frac{1 \times 4}{5 \times 4} + \frac{1 \times 4}{5 \times 4} - 2 \times \frac{1}{6_{3}}$

$= \frac{5}{20} + \frac{4+4}{20} - \frac{1}{3}$

$= \frac{13 \times 3}{20 \times 3} - \frac{1 \times 20}{3 \times 20}$

$= \frac{39 - 20}{60}$

$39 - 20 = 19$

$= \frac{19}{60}$

Q) A box contain 6 red ball, 7 white ball 3 green balls. if a ball is drawn at random, probability that it is red or green

A → red 6/16

B → green 3/16

$P(A \cup B) = \frac{6+3}{16} = 9/16$

1. Probability that a student passes in an exam is 0.8. Probability that another student passes 0.7. what is the probability that one of them would pass.

$P(A) \Rightarrow 0.8$ $\quad\quad$ $P(\bar{A}) = 1 - 0.8 = 0.2$

$P(B) \Rightarrow 0.7$. $\quad\quad$ $P(\bar{B}) = 1 - 0.7 = 0.3$.

$\cancel{P(A \cap \bar{B}) \cup P(A}$

$$P[(A \cap \bar{B}) \cup (\bar{A} \cap B)] =$$

$$= P(A) \cdot P(\bar{B}) + P(\bar{A}) \cdot P(B)$$